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Quantum radiative corrections to slow-roll inflation

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We consider the nonminimally coupled $\lambda\varphi^4$ scalar field theory in de Sitter space and construct the renormalization group improved renormalized effective theory at the one-loop level. Based on the corresponding quantum Friedmann equation and the scalar field equation of motion, we calculate the quantum radiative corrections to the scalar spectral index n_s, gravitational wave spectral index n_g and the ratio r of tensor to scalar perturbations. When compared with the standard (tree-level) values, we find that the quantum contributions are suppressed by λN^2 where N denotes the number of e-foldings. Hence there is an N^2 enhancement with respect to the naïve expectation, which is due to the infrared enhancement of scalar vacuum fluctuations characterising de Sitter space. Since observations constrain λ to be very small $\lambda \sim 10^{-12}$ and $N \sim 50-60$, the quantum corrections in this inflationary model are unobservably small.

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1. INTRODUCTION

Given the fact that we live in the era of precision cosmology, it is important to establish a framework within which the quantum radiative corrections for observables induced by the vacuum fluctuations of matter fields can be calculated. Such radiative corrections may be important in some inflationary models. In this paper we consider the nonminimal $\lambda\varphi^4$ inflationary model which includes a nonvanishing coupling ξ to the curvature scalar. We consider this model for simplicity; once the framework for calculating the quantum (radiative) corrections is established, it can be

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quite easily applied to other inflationary models [1].

The minimally coupled $\lambda\varphi^4$ inflationary model (with $\xi = 0$) is already more than two standard deviations disfavored by cosmological observations [2]. However, for a certain choice of the coupling ξ to the background, the resulting nonminimally coupled scalar field model can still match the experimental data [3, 4, 5, 6], basically because the model then produces the spectral index of the massive chaotic inflaton model. In this work we show that this is indeed the case, but only for a rather limited values of ξ, namely for ξ which satisfies

$$\frac{1}{8\tilde{N}} \ll |\xi| \ll \frac{1}{24}\,, \qquad (\xi < 0)\,, \tag{1}$$

where $\tilde{N} \simeq N + 1$ and N is the number of e-foldings.

If the condition $|dH/dt| \ll H^2$ is not fulfilled, then our framework is not applicable, because we have constructed the de Sitter invariant scalar field propagator with the assumption that the Ricci scalar $R = 6(2H^2 + dH/dt) \simeq 12H^2$, where $H = H(t)$ denotes the Hubble parameter. This condition is fulfilled in most of inflationary models and thus does not present a significant constraint to our model.

During inflation the amplitude of field correlators at the classical level is suppressed by powers of the Hubble parameter, but the quantum corrections to field correlators can depend on the whole history of inflation, leaving hence the possibility that in a cumulative manner quantum corrections can become important and even detectable by future experiments [7, 8, 9, 10, 11]. Such cumulative effects are claimed to be present in the analysis done recently in Refs. [12, 13]. In our analysis no such cumulative effects are present.

The quantum radiative corrections to slow-roll parameters in inflation have been firstly calculated in Refs. [7, 8, 9]. The authors begin by considering single field inflationary models and subsequently generalize their analysis to include the inflaton coupling to a light scalar and light fermionic field. While Refs. [7, 8] consider quantum corrections to the equation of motion in momentum space, in this work we make use of the effective action techniques. Our results are in a qualitative disagreement with those of Refs. [7, 8, 9]. One important difference is in that in their analysis the authors of [7, 8, 9] neglect the inflaton coupling to the background curvature (Ricci scalar), which within our framework yields the dominant contribution to the quantum radiative corrections during slow-roll inflation. A second important difference is that we made our analysis by using the de Sitter invariant propagator, while the proper analysis should be conducted by making use of a scalar propagator suitable for quasi-de Sitter spaces. Our method is based on the effective action approach. We arrive at our one-loop effective action by making use of the position space propagator at coincidence.

Within this method we are able to use the well established machinery of dimensional regularization, renormalization and renormalization group improvement of our resulting effective field theory. The authors of Ref. [9] advocate the use of the dynamical renormalisation group method (DRG) [14]. In that novel method the secular terms, which induce a logarithmic growth (with conformal time) of the mode functions, are resummed to yield the renormalisation group improved mode functions. These improved mode functions exhibit regulated late time infrared divergences, rendering the mode functions infrared finite.

More specifically, within our framework we obtain a quantum infrared enhancement to slow-roll parameters which is, when compared to the classical values, proportional to the number of e-foldings squared N^2. This enhancement is due to the scalar field mass generated by the coupling to the background curvature scalar. The authors of [7, 8, 9] obtain a quadratic enhancement but for the $\lambda\varphi^4$ model without including the inflaton coupling to the Ricci scalar. On the other hand, when compared to the classical value, the quantum corrections generated by the $\lambda\varphi^4$ interaction term are enhanced in our framework only linearly by N.

It is by now a well established fact that quantum effects can have quite a dramatic impact during inflation. An example is the breakdown of conformal invariance for the photons of scalar electrodynamics, which has as a consequence a photon mass generation during inflation and a generation of cosmological scale magnetic fields [15]. Similarly the quantum radiative effects break conformal invariance of the fermions of the Yukawa theory in de Sitter space [16]. As a result fermions acquire a mass during inflation [16, 17], having as a consequence a production of fermions during inflation and possibly inflationary baryogenesis. Finally, the canonical coupling of gravitons to fermions enhances the production of fermions during inflation [18].

The main result of our work is the quantum correction to the scalar spectral index (107–108). The leading order contribution reads

$$(n_s-1)_Q = \frac{\lambda\tilde{N}(\xi-1/6)}{18\pi^2}\frac{\kappa}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2} + \mathcal{O}(\lambda\ln(\tilde{N}))\,, \qquad (\kappa = 8\tilde{N}\xi, \tilde{N} \approx N+1-\xi/2)\,, \quad (2)$$

which is to be compared with the classical contribution,

$$(n_s-1)_C = -\frac{3}{\tilde{N}}\frac{1-\frac{2}{3}\kappa}{1-\kappa}\,. \qquad (3)$$

The leading contribution (2) originates solely from the resummation of the mass insertions generated by the coupling to the background curvature. Since $\lambda \sim 10^{-12}$ the quantum contribution (2) is indeed too small to be observable. Note that the condition (98) implies $-\tilde{N}/3 \ll \kappa < 1$, such that κ can be large and negative. If this is the case the classical spectral index (3) becomes consistent

with the current CMB data [2]. We futhermore calculate the quantum corrections to the spectrum of curvature perturbation, to the tensor spectral index, and to the ratio of the tensor-to-scalar spectrum.

The present work is organized as follows. In Section 2 we first recall the basics of de Sitter space and then sketch the derivation of the de Sitter invariant Chernikov-Tagirov propagator. In Section 3 we use this propagator and the techniques of dimensional regularization and renormalization to derive the one-loop improved effective potential for our theory. This procedure requires one counterterm for the quartic self-coupling constant λ and one for the coupling to the background ξ, which are defined at an arbitrary scale φ_0. In Section 4 we use the standard renormalization group (RG) techniques to improve our effective potential. Having obtained the RG improved effective potential, in Section 5 we calculate the corresponding quantum scalar field stress-energy tensor in the slow-roll approximation. By making use of the quantum Friedmann equation and of the scalar field equation of motion, we then develop our framework within which the quantum radiative corrections from the vacuum matter fluctuations to slow-roll parameters can be calculated. In particular, we organize the quantum radiative corrections to slow-roll parameters ϵ and η into two distinct parts. The first part arises from the one-loop resummation of the mass insertions generated by the quartic self-coupling in the presence of a scalar (inflaton) condensate, while the second part arises from the resummation of the scalar mass insertions generated by the coupling to the background. Both of these quantum corrections are suppressed by the coupling constant λ but they are enhanced by the number of e-foldings squared. Based on these results we then calculate the quantum radiative corrections to the observables: the spectrum of curvature perturbation and its spectral index, the tensor spectral index and the ratio of tensor-to-scalar spectra. Finally, in Section 6 we summarize our results and discuss their physical implications.

2. PROPAGATOR IN DE SITTER SPACE

2.1. de Sitter space

A four dimensional de Sitter space is perhaps best viewed as a 4-dimensional hyperboloid embedded into the 5-dimensional Minkowski space-time with the line element,

$$ds_5^2 = -dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2 \,. \tag{4}$$

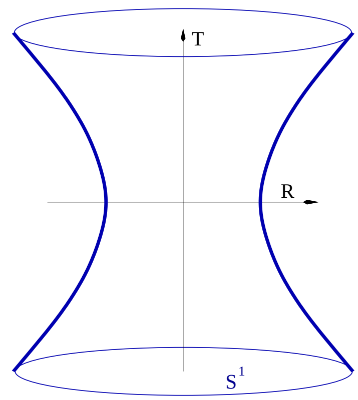

FIG. 1: The embedding of de Sitter space into a five dimensional flat space-time. The vertical line corresponds to the time coordinate, $X_0 = T$, and the radial coordinate $R = \sqrt{X_1^2 + X_2^2 + X_3^2 + X_4^2}$. At each point (T, R) there is a unit 3-sphere S^3, which is for the sake of clarity represented by a circle S^1 erected at each point (T, R). The Hubble radius $R_H = 1/H$ is the coordinate distance R of the hyperboloid from the origin at $T = 0$.

The embedded hyperboloid of de Sitter space is shown in Figure 1, and it is determined by

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \frac{1}{H^2}\,, \tag{5}$$

where H denotes the Hubble parameter. The symmetry group of de Sitter space, $SO(1,4)$, is manifest by this embedding. One defines the de Sitter invariant distance functions as,

$$Z(X;X') = H^2 \sum_{A,B=0}^{4} \eta_{AB} X_A X'_B = 1 - \frac{1}{2} Y(X;X')\,, \qquad \eta_{AB} = \mathrm{diag}(-1,1,1,1,1)\,. \tag{6}$$

We shall use the following flat 4-dimensional coordinates (which cover 1/2 of the de Sitter manifold),

$$\begin{aligned}
X_0 &= \frac{1}{H}\sinh(Ht) + \frac{H}{2}\,\mathrm{e}^{Ht}\|\vec{x}\|^2\,, \qquad && (-\infty < t < \infty)\,,\\
X_i &= \mathrm{e}^{Ht} x_i\,, && (-\infty < x_i < \infty,\ i = 1,2,3)\,,\\
X_4 &= \frac{1}{H}\cosh(Ht) - \frac{H}{2}\,\mathrm{e}^{Ht}\|\vec{x}\|^2\,,
\end{aligned} \tag{7}$$

in which the metric tensor reduces to the form

$$ds^2 = -dt^2 + a^2 d\vec{x}^{\,2}\,, \tag{8}$$

with the scale factor $a = \mathrm{e}^{Ht}$. When written in terms of conformal time η, defined as $a d\eta = dt$, the metric tensor acquires the conformal form,

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}\,, \qquad a = -\frac{1}{H\eta} \quad (\eta < 0)\,, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)\,. \tag{9}$$

The invariant distance functions $Z(X;X') \equiv z(x;x')$ and $Y(X;X') \equiv y(x;x')$ reduce in these coordinates to the simple form,

$$z(x;x') = 1 - \frac{1}{2}y(x;x')\,, \qquad y(x;x') = aa'H^2\Delta x^2\,, \tag{10}$$

with $a = a(\eta) = -1/(H\eta)$, $a' = a(\eta') = -1/(H\eta')$, and

$$\Delta x^2(x;x') = -(|\eta - \eta'| - i\varepsilon)^2 + \|\vec{x} - \vec{x}\,'\|^2\,, \tag{11}$$

where (for a later use) we introduced the infinitesimal parameter $\varepsilon > 0$, which defines how the poles of the propagator (discussed in the next section) contribute. In these coordinates the curvature of spatial sections vanishes, and thus they are also known as flat (Euclidean) coordinates, in which de Sitter space appears as uniformly expanding.

By solving the relevant geodesic equations for x^0 and x^i $(i = 1,2,3)$ one can show that the de Sitter invariant distance function $y = y(x;x'\,)$ is related to the geodesic distance $\ell = \ell(x;x'\,)$ by the following simple relation,

$$y(x;x'\,) = 4\sin^2\left(\frac{1}{2}H\ell(x;x'\,)\right)\,. \tag{12}$$

2.2. Scalar propagator in de Sitter space

The dynamics of the scalar field are specified by the following tree-level action,

$$S_\varphi = \int d^4x\sqrt{-g}\mathcal{L}_\varphi\,, \tag{13}$$

with the Lagrangean,

$$\sqrt{-g}\mathcal{L}_\varphi = \sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - V_b(\varphi)\Big)\,, \tag{14}$$

where

$$V_b(\varphi) = \frac{1}{2}\xi_b\mathcal{R}\varphi^2 + \frac{\lambda_b}{4!}\varphi^4\,. \tag{15}$$

In the above expression λ_b and ξ_b are the bare values of the quartic self-coupling and the nonminimal coupling to the Ricci curvature scalar $\mathcal{R}$, respectively, and $g = \det(g_{\mu\nu})$. For simplicity we set the bare scalar mass $m_b = 0$. The theory (13–15) is a simplified version of the Yukawa theory in de Sitter background studied in Refs. [20**?**]. An early related work can be found in Ref. [19].

The scalar propagator in a curved background space can be defined as the expectation value,

$$i\Delta(x;x') = \langle x|\frac{i}{\sqrt{-g}(\Box - m_\varphi^2 - \xi_b\mathcal{R}_D)}|x'\rangle\,, \qquad \Big(m_\varphi^2(\varphi) \equiv \frac{\lambda_b}{2}\varphi^2\Big)\,, \tag{16}$$

where $|x\rangle$ is the eigenstate of the position operator $\hat{x}$ (*i.e.* $\hat{x}|x\rangle = x|x\rangle$), $\Box = (-g)^{-1/2}\partial_\mu(-g)^{1/2}g^{\mu\nu}\partial_\nu$ denotes the d'Alambertian and $g = \det[g_{\mu\nu}]$. This Feynman propagator solves the following equation in de Sitter space in general D space-time dimensions (needed for dimensional regularization and renormalization),

$$\sqrt{-g}(\Box - m_\varphi^2 - \xi_b \mathcal{R}_D) i\Delta(x;x') = i\delta^D(x-x')\,, \tag{17}$$

where $\delta^D(x-x')$ is the D-dimensional Dirac δ-distribution, $\mathcal{R}_D = D(D-1)H^2$ is the Ricci scalar in a D dimensional de Sitter space, and H is the Hubble parameter.

The de Sitter invariant form of (17) is [22, 23, 24, 25]

$$\Big[(1-z^2)\frac{d^2}{dz^2} - Dz\frac{d}{dz} - \frac{m_\varphi^2 + \xi_b R_D}{H^2}\Big] iG(y) = \frac{i\delta^D(x-x')}{H^2 a^D}\,, \tag{18}$$

where the invariant propagator is defined as, $iG(y) = i\Delta(x;x')$. Here we made use of Eqs. (10–11) and of

$$\partial_\mu \equiv (\partial_\mu z)\frac{d}{dz} = -\frac{1}{2}Ha\Big(\delta_\mu^{\,0} y + 2a' H\Delta x_\mu\Big)\frac{d}{dz}\,. \tag{19}$$

The properly normalized de Sitter invariant solution of Eq. (18), which near the light-cone and in the massless limit reduces to the Hadamard form,

$$iG(y) \;\overset{y\rightarrow 0}{\longrightarrow}\; \frac{H^{D-2}}{(2\pi)^{D/2}}\Gamma\Big(\frac{D}{2}-1\Big)\frac{1}{y^{\frac{D}{2}-1}} + \mathcal{O}\Big(y^{2-D/2}, y^0\Big)\,, \tag{20}$$

is unique,

$$iG(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\Big(\frac{D-1}{2}+\nu_D\Big)\Gamma\Big(\frac{D-1}{2}-\nu_D\Big)}{\Gamma\Big(\frac{D}{2}\Big)}\,{}_2F_1\Big(\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y}{4}\Big)\,, \tag{21}$$

where

$$\nu_D = \Big(\frac{(D-1)^2}{4} - \frac{m_\varphi^2 + \xi_b R_D}{H^2}\Big)^{\frac{1}{2}}\,. \tag{22}$$

This is the Chernikov-Tagirov propagator for de Sitter space [22, 23] generalized to D space-time dimensions. The pole prescription defined by the $i\varepsilon$-prescription in Eq. (11) implies that the propagator (21) corresponds to the time-ordered (Feynman) propagator. For a discussion of other propagators relevant for expanding space-times in the Schwinger-Keldysh *in-in* formalism we refer to [10, 25].

3. EFFECTIVE POTENTIAL

The one-loop effective action for a real scalar field reads,

$$\Gamma_\varphi[g_{\mu\nu},\varphi] \equiv \int d^Dx\sqrt{-g}\mathcal{L}_\varphi = S_{\rm HE}[g_{\mu\nu}] + S_\varphi[g_{\mu\nu},\varphi] + \frac{i}{2}{\rm Tr}\ln\Big(\sqrt{-g}(\Box - m_\varphi^2 - \xi_b\mathcal{R}_D)\Big)\,, \tag{23}$$

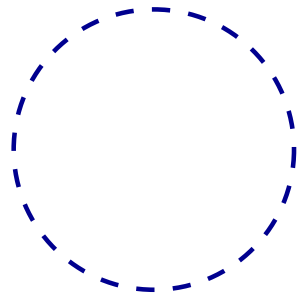

FIG. 2: The one-loop diagram (vacuum bubble) contributing to the scalar effective theory (23) in a curved background.

where Tr refers to the space-time integration $\int d^D x$, S_φ is the tree-level scalar field action (13) and $S_{\rm HE}$ denotes the Hilbert-Einstein action,

$$S_{\rm HE} = -\frac{1}{16\pi G_N}\int d^D x\sqrt{-g}\mathcal{R}_{\mathcal{D}}\,. \tag{24}$$

The last term in Eq. (23) represents the one-loop contribution to the effective action $\delta_1\mathcal{L}_\varphi$, whose graphical representation is shown in Figure 2. We shall now evaluate the general expression (23) in de Sitter space. It is convenient to differentiate the one-loop contribution $\delta_1\mathcal{L}_\varphi$ with respect to the scalar mass,

$$\frac{\partial\delta_1\mathcal{L}_\varphi}{\partial m_\varphi^2} = \frac{i}{2}\Big\langle x\Big|\frac{-1}{\sqrt{-g}(\Box - m_\varphi^2 - \xi_b\mathcal{R}_D)}\Big|x\Big\rangle = -\frac{1}{2}\,i\Delta(x;x)\,. \tag{25}$$

Now making use of Eq. (21) one obtains,

$$\frac{\partial\delta_1\mathcal{L}_\varphi}{\partial m_\varphi^2} = -\frac{1}{2}iG(y)|_{y\to0} = -\frac{1}{2}\frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\frac{\Gamma\Big(\frac{D-1}{2}+\nu_D\Big)\Gamma\Big(\frac{D-1}{2}-\nu_D\Big)}{\Gamma\Big(\frac{1}{2}+\nu_D\Big)\Gamma\Big(\frac{1}{2}-\nu_D\Big)}\,. \tag{26}$$

Separating the divergent and finite contributions in (26) yields

$$\begin{aligned}\frac{\partial\delta_1\mathcal{L}_\varphi}{\partial m_\varphi^2} = & -\frac{1}{2}\frac{H^{D-2}}{(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\Big(\frac{m_\varphi^2}{H^2} - (D-2) + \xi_b D(D-1)\Big) \\ & -\frac{H^2}{32\pi^2}\Big(\frac{m_\varphi^2}{H^2} - 2(1-6\xi_b)\Big)\Big[\psi\Big(\frac{1}{2}+\nu\Big) + \psi\Big(\frac{1}{2}-\nu\Big)\Big]\,,\end{aligned} \tag{27}$$

where we made use of,

$$\Gamma\Big(1-\frac{D}{2}\Big) = \frac{2}{D-4} - (1-\gamma_E) + \mathcal{O}(D-4)\,, \tag{28}$$

$\gamma_E \simeq 0.577$ is the Euler constant, $\mathcal{R}_{D=4} \equiv \mathcal{R} = 12H^2$, and

$$\nu = \Big(\frac{1}{4} - \frac{m_\varphi^2}{H^2} + 2(1-6\xi_b)\Big)^{\frac{1}{2}}\,, \tag{29}$$

where $m_\varphi^2 = \lambda\varphi^2/2$ is defined in Eq. (16). When integrated, Eq. (27) gives the following contribution to the effective Lagrangean,

$$\begin{aligned}\delta_1\mathcal{L}_\varphi &= -\frac{1}{2(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\Big(\frac{1}{2}H^{D-4}m_\varphi^4 - \Big[(D-2) - \xi_b D(D-1)\Big]H^{D-2}m_\varphi^2\Big) \\ &\quad -\frac{H^2}{32\pi^2}\int dww\Big[\psi\Big(\frac{1}{2}+\nu\Big)+\psi\Big(\frac{1}{2}-\nu\Big)\Big]\,, \qquad w = \frac{m_\varphi^2}{H^2} + 12\left(\xi_b - \frac{1}{6}\right)\,,\end{aligned} \tag{30}$$

where the integral is an indefinite integral.

In order to renormalize our Lagrangean $\mathcal{L}_\varphi$ we will add to it the counterterms λ_0 and ξ_0 and apply the renormalization conditions which will determine the finite parts of those counterterms,

$$\mathcal{L}_{\varphi,\,\mathrm{ren}} = -\frac{1}{2}g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - V_{\mathrm{ren}}(\varphi)\,, \tag{31}$$

where now

$$V_{\mathrm{ren}}(\varphi) = \frac{\lambda_b}{4!}\varphi^4 + \frac{1}{2}\xi_b\mathcal{R}\varphi^2 + \frac{\lambda_0}{4!}\varphi^4 + \frac{1}{2}\xi_0\mathcal{R}\varphi^2 - \delta_1\mathcal{L}_\varphi, \tag{32}$$

and $\delta_1\mathcal{L}_\varphi$ is given by Eq. (30).

We renormalize our Lagrangean at an arbitrary scale φ_0,

$$\begin{aligned}\frac{\delta^4 V}{\delta\varphi^4}\bigg|_{\varphi=\varphi_0} &= \lambda_b = \lambda_b + \lambda(\varphi_0,H^2) - \frac{\delta^4(\delta_1\mathcal{L})}{\delta\varphi^4}\bigg|_{\varphi=\varphi_0}\,, \\ \frac{\delta^3 V}{\delta(H^2)\delta\varphi^2}\bigg|_{\varphi=\varphi_0} &= D(D-1)\xi_b = D(D-1)\xi_b + D(D-1)\xi(\varphi_0,H^2) - \frac{\delta^3(\delta_1\mathcal{L})}{\delta(H^2)\delta\varphi^2}\bigg|_{\varphi=\varphi_0}\,,\end{aligned} \tag{33}$$

which yields

$$\begin{aligned}\lambda(\varphi_0,H^2) &= -\frac{3}{2(4\pi)^{D/2}}\Gamma(1-\frac{D}{2})\mu^{D-4}\lambda_b^2 \\ &\quad -\frac{3\lambda_b^2}{32\pi^2}\left[\ln\left(\frac{\lambda_b\varphi_0^2 + 24H^2(\xi_b-1/6)}{2\mu^2}\right) + \frac{7}{3} + \frac{4}{3}\frac{\lambda_b\varphi_0^2}{\lambda_b\varphi_0^2 + 24H^2(\xi_b-1/6)}\right]\,, \\ \xi(\varphi_0,H^2) &= \frac{\lambda_b}{2(4\pi)^{D/2}}\Gamma(1-\frac{D}{2})\left(\frac{D-2}{D(D-1)} - \xi_b\right)\mu^{D-4} \\ &\quad -\frac{\lambda_b}{32\pi^2}\left[\left(\xi_b - \frac{1}{6}\right)\ln\left(\frac{\lambda_b\varphi_0^2 + 24H^2(\xi_b-1/6)}{2\mu^2}\right) + 3\left(\xi_b-\frac{1}{6}\right) - \frac{1}{36}\right]\,.\end{aligned} \tag{34}$$

From (33) it follows that the counterterms λ_0 and ξ_0 are given by

$$\begin{aligned}\lambda_0 \equiv \lambda(\varphi_0,0) &= -\frac{3}{2(4\pi)^{D/2}}\Gamma\left(1-\frac{D}{2}\right)\mu^{D-4}\lambda_b^2 - \frac{3\lambda_b^2}{32\pi^2}\left[\ln\left(\frac{\lambda_b\varphi_0^2}{2\mu^2}\right) + \frac{11}{3}\right] \\ \xi_0 \equiv \xi(\varphi_0,0) &= \frac{\lambda_b}{2(4\pi)^{D/2}}\Gamma\Big(1-\frac{D}{2}\Big)\Big[\frac{D-2}{D(D-1)} - \xi_b\Big]\mu^{D-4} \\ &\quad -\frac{\lambda_b}{32\pi^2}\Big[\Big(\xi_b-\frac{1}{6}\Big)\ln\Big(\frac{\lambda_b\varphi_0^2}{2\mu^2}\Big) + 3\Big(\xi_b-\frac{1}{6}\Big) - \frac{1}{36}\Big]\,.\end{aligned} \tag{35}$$

Given that H and φ are dynamical fields, the counterterm parameters λ_0 and ξ_0 must be independent of H and φ, which is indeed satisfied by (35). Now making use of Eqs. (30), (32) and (35) we can calculate the renormalized Lagrangean (31). The result is,

$$\begin{aligned}\mathcal{L}_{\varphi,\,\mathrm{ren}} &= -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\varphi^4}{4!}\Big\{\lambda_b + \frac{3\lambda_b^2}{32\pi^2}\Big[\ln\Big(\frac{2H^2}{\lambda_b\varphi_0^2}\Big) - \frac{11}{3}\Big]\Big\}\\ &-\frac{1}{2}\mathcal{R}\varphi^2\Big\{\xi_b + \frac{\lambda_b}{32\pi^2}\Big[\Big(\xi_b-\frac{1}{6}\Big)\ln\Big(\frac{2H^2}{\lambda_b\varphi_0^2}\Big) - 3\Big(\xi_b-\frac{1}{6}\Big)+\frac{1}{36}\Big]\Big\}\\ &-\frac{H^4}{32\pi^2}\int dww\Big[\psi\Big(\frac{1}{2}+\nu\Big)+\psi\Big(\frac{1}{2}-\nu\Big)\Big]\,. \end{aligned} \tag{36}$$

This is the fully renormalized effective Lagrangean. We now consider the two asymptotic forms of (36), first the ultraviolet (UV) limit.

The following asymptotic expansion of the di-gamma function, $\psi(z) = (d/dz)[\ln(\Gamma(z))]$, is then useful (*cf.* Eq. (8.344) in [26]),

$$\psi(z) = \ln(z) - \frac{1}{2z} - \frac{1}{12z^2} + \frac{1}{120z^4} + \mathcal{O}(z^{-6})\,, \tag{37}$$

such that

$$\psi\Big(\frac{1}{2}+\nu\Big)+\psi\Big(\frac{1}{2}-\nu\Big) = \ln(w) - \frac{1}{3w} - \frac{1}{15w^2} + \mathcal{O}(1/w^3)\,, \qquad \nu^2 = \frac{1}{4} - w\,. \tag{38}$$

Upon evaluating the integral in (27) one obtains

$$-\frac{H^4}{32\pi^2}\Big\{\frac{w^2}{2}\Big[\ln(w)-\frac{1}{2}\Big] - \frac{w}{3} - \frac{1}{15}\ln(w) + \mathcal{O}(1/w)\Big\}\,. \tag{39}$$

Taking account of this, we can recast Eq. (36) to the form,

$$\begin{aligned}\mathcal{L}_{\varphi} &= -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\varphi^4}{4!}\left\{\lambda_b + \frac{3\lambda_b^2}{32\pi^2}\left[\ln\left(\frac{\lambda_b\varphi^2+24H^2\left(\xi_b-\frac{1}{6}\right)}{\lambda_b\varphi_0^2}\right) - \frac{25}{6}\right]\right\}\\ &-\frac{1}{2}\mathcal{R}\varphi^2\left\{\xi_b + \frac{\lambda_b}{32\pi^2}\left[\left(\xi_b-\frac{1}{6}\right)\ln\left(\frac{\lambda_b\varphi^2+24H^2\left(\xi_b-\frac{1}{6}\right)}{\lambda_b\varphi_0^2}\right) - \frac{7}{2}\left(\xi_b-\frac{1}{6}\right)+\frac{1}{72}\right]\right\}\\ &-\frac{\mathcal{R}^2}{64\pi^2}\left\{\left(\xi_b-\frac{1}{6}\right)^2\left[\ln\left(\frac{\lambda_b\varphi^2}{2H^2}+12\left(\xi_b-\frac{1}{6}\right)\right)-\frac{1}{2}\right] - \frac{1}{18}\left(\xi_b-\frac{1}{6}\right)\right.\\ &\left.-\frac{1}{1080}\ln\left(\frac{\lambda_b\varphi^2}{2H^2}+12\left(\xi_b-\frac{1}{6}\right)\right)\right\} + \mathcal{O}(\mathcal{R}^3). \end{aligned} \tag{40}$$

This is the UV form of (36). In the limit $H^2 \to 0$ to flat Minkowski spacetime, the first line in (40) reproduces the classical Coleman-Weinberg result [27].

To complete the analysis of the effective Lagrangean (36) we still need to consider the small field limit of the integral in (36). Let us first consider the expansion which is applicable around the poles

of the di-gamma function, which are located at

$$\nu_n = \frac{1}{2} + n\,, \qquad n = 0, 1, 2, 3, \ldots\,. \tag{41}$$

This implies that the poles are located at

$$w_n = -n(n+1) \tag{42}$$

and in the vicinity of the poles we can write

$$w = w_n + \delta w\,. \tag{43}$$

With the above definitions, a conformally coupled scalar field with $\xi = 1/6$ corresponds to $n = 0$ and a minimally coupled scalar field with $\xi = 0$ corresponds to $n = 1$. More generally we have,

$$\xi_n = \frac{(1-n)(2+n)}{12}\,, \qquad (n \geq 0)\,, \tag{44}$$

such that for $n > 1$ all $\xi_n < 0$. In particular, $\xi_2 = -1/3$, $\xi_3 = -5/6$, *etc.* Now we can expand ν as

$$\nu = \nu_n - \frac{\delta w}{2\nu_n} - \frac{1}{8}\frac{(\delta w)^2}{\nu_n^3} \tag{45}$$

and by making use of

$$\psi\Big(\frac{1}{2} - \nu\Big) = \psi\Big(n + \frac{3}{2} - \nu\Big) - \sum_{\ell=0}^{n} \frac{1}{\frac{1}{2} + \ell - \nu} \tag{46}$$

we can finally write

$$w\left[\psi\Big(\frac{1}{2} + \nu\Big) + \psi\Big(\frac{1}{2} - \nu\Big)\right] = \frac{a_n}{\delta w} + b_n + c_n \delta w + \mathcal{O}((\delta w)^2)\,. \tag{47}$$

In the above expressions

$$\begin{aligned}
a_n &= n(n+1)(2n+1)\,, \\
b_n &= -2n(n+1)\psi(n+1) - (2n+1) - \frac{n(n+1)}{2n+1}\,, \\
c_n &= \frac{1}{2n+1} + 2\psi(n+1) + \frac{2n(n+1)}{2n+1}\left(-\psi'(1) + \psi'(n+1) - \frac{1}{2(2n+1)^2}\right)\,,
\end{aligned} \tag{48}$$

and we made use of

$$\sum_{\ell=1}^{n} \frac{1}{\ell} = \psi(n+1) - \psi(1)\,, \quad \sum_{\ell=1}^{n} \frac{1}{\ell^2} = -\psi'(n+1) + \psi'(1)\,, \tag{49}$$

where $\psi(1) = -\gamma_E = 0.577215...$ is the Euler constant, $\psi'(1) = \pi^2/6$ and $\psi'(z+1) = \psi'(z) - 1/z^2$.

We can now write the infrared limit of the renormalized Lagrangean (36). From Eqs. (47–48) it follows that the integral in the last line of Eq. (36) has the infrared limit,

$$-\frac{H^4}{32\pi^2}\left[a_n\ln(\delta w_n)+b_n(\delta w_n)+\frac{1}{2}c_n(\delta w_n)^2+\mathcal{O}\Big((\delta w_n)^3\Big)\right], \tag{50}$$

where

$$\delta w_n=\frac{\lambda\varphi^2}{2H^2}+12\delta\xi_n\,,\qquad \delta\xi_n=\xi_b+\frac{1}{12}(n-1)(n+2)\ll 1 \tag{51}$$

and a_n, b_n and c_n are defined in Eq. (48).

4. RENORMALIZATION GROUP ANALYSIS

In deriving expression (36) we have introduced an arbitrary renormalization scale φ_0 by defining the renormalization conditions (33). From this definition an arbitrary scale φ_0 enters into the expressions for the counterterms (35) and hence into the renormalized Lagrangean (36). However, as it was stressed in the classic paper of Coleman and Weinberg in 1973, the change of the renormalization scale can only change the definitions of coupling constants, not the physics [27].

By applying the same reasoning in our case, we arrive at the following conclusion: a small change in φ_0 in the expression for the physical quantity of interest can always be compensated for by an appropriate small change in λ and ξ. The convenient way of expressing this statement is

$$\left(\varphi_0\frac{\partial}{\partial\varphi_0}+\beta_\lambda\frac{\partial}{\partial\lambda}+\beta_\xi\frac{\partial}{\partial\xi}\right)V_{\rm eff}(\varphi_0,\lambda,\xi,\varphi)=0, \tag{52}$$

which is the standard Callan-Symanzik (CS) equation written for the theory at hand. The renormalization group functions β_λ and β_ξ are given by the following relations,

$$\begin{aligned}\beta_\lambda &= -\varphi_0\left.\frac{\partial\lambda_0}{\partial\varphi_0}\right|_{\lambda_b},\\ \beta_\xi &= -\varphi_0\left.\frac{\partial\xi_0}{\partial\varphi_0}\right|_{\xi_b}.\end{aligned} \tag{53}$$

Within the one-loop approximation the renormalization group functions β_λ and β_ξ are uniquely determined as the coefficients of the divergent logarithmic terms appearing in the counterterms λ_0 and ξ_0 (35). It follows (writing λ_b and ξ_b from now on as λ and ξ, respectively):

$$\begin{aligned}\beta_\lambda &= \frac{3\lambda^2}{16\pi^2},\\ \beta_\xi &= \frac{\lambda}{16\pi^2}\left(\xi-\frac{1}{6}\right).\end{aligned} \tag{54}$$

These expressions for β_λ and β_ξ we will use to determine the running of λ and ξ with the scale φ_0.

We shall now solve the Callan-Symanzik equation (52). From the theory of partial differential equations we can make use of the method of characteristics [29, 30, 31, 32]. Applying this method to (52) we can write down the solution to the Callan-Symanzik equation (52) as

$$V_{\rm eff}(\varphi_0,\lambda,\xi,\varphi) = V_{\rm eff}(\varphi_0(t),\lambda(t),\xi(t),\varphi(t)), \tag{55}$$

where $\varphi_0(t),\lambda(t),\xi(t),\varphi(t)$ are the running parameters. The t-dependence of the running parameters is given (to the order we are working in) by the following differential equations:

$$\begin{aligned} \frac{d\varphi_0(t)}{dt} &= \varphi_0(t), & \frac{d\varphi(t)}{dt} &= 0, \\ \frac{d\lambda(t)}{dt} &= \beta_\lambda(\lambda(t)), & \frac{d\xi(t)}{dt} &= \beta_\xi(\xi(t),\lambda(t)). \end{aligned} \tag{56}$$

The boundary conditions (at $t=0$) are,

$$\begin{aligned} \varphi_0(0) &= \varphi_0, & \varphi(0) &= \varphi, \\ \lambda(0) &= \lambda, & \xi(0) &= \xi. \end{aligned} \tag{57}$$

The solutions of the first two differential equations in (56) are trivial and read,

$$\varphi_0^2(t) = \varphi_0^2 e^{2t}, \qquad \varphi(t) = \varphi\,. \tag{58}$$

When combined with the previous results (54), the last two differential equations in (56) are solved by

$$\lambda(t) = \frac{\lambda}{1-\frac{3\lambda}{16\pi^2}t}, \tag{59}$$

and

$$\xi(t) = \frac{1}{6} + (\xi - \frac{1}{6})\left(\frac{\lambda(t)}{\lambda}\right)^{1/3}, \tag{60}$$

where here $\lambda = \lambda(0)$ and $\xi = \xi(0)$. These solutions imply that our model exhibits an infrared fixed point, where the coupling constants are $\lambda_{\rm FP} = 0$ and $\xi_{\rm FP} = 1/6$. In this limit the theory possesses an enhanced symmetry (conformal symmetry) and it can be reduced to a pure metric theory (see Ref. [33] for a nonperturbative proof of this statement).

We stress that the parameter t in the above relations is completely arbitrary. The basic idea of the renormalization group (RG) improvement of an effective potential is that we can choose t in such a way that the perturbation series for the effective potential converges more rapidly. Indeed by suitably choosing t one can extend the range of validity of the effective theory to a larger range of the dynamical quantities H and φ by replacing the perturbative expression on the left-hand side of (55) by its right-hand side. This is intimately related to the fact that the unimproved expression

for the effective potential is actually valid only for φ's sufficiently close to φ_0. Since the change of the arbitrary scale φ_0 corresponds just to a reparametrization of the coupling constants within our theory, the unimproved effective potential is valid only near $\varphi \sim \varphi_0$ and $H \sim 0$ and thus not a very useful quantity. If we, on the other hand, require that the effective theory does not depend on φ_0, then the improved effective potential (which solves the Callan-Symanzik equation) remains valid whenever the coupling constants are small.

Since the perturbation series for the effective potential is characterized by the occurrence of powers of logarithmic terms, we choose[1]

$$t = \ln\left(\frac{\varphi}{\varphi_0}\right) . \tag{61}$$

The improved expression for the renormalized effective potential now becomes

$$\begin{aligned} V_{\rm RG}(\varphi) \;=\; & \frac{\varphi^4}{4!}\Big\{\lambda + \frac{3\lambda^2}{32\pi^2}\Big[\ln\Big(\frac{2H^2}{\lambda\varphi^2}\Big) - \frac{11}{3}\Big]\Big\} \\ & + 6H^2\varphi^2\Big\{\xi + \frac{\lambda}{32\pi^2}\Big[\Big(\xi - \frac{1}{6}\Big)\ln\Big(\frac{2H^2}{\lambda\varphi^2}\Big) - 3\Big(\xi - \frac{1}{6}\Big) + \frac{1}{36}\Big]\Big\} \\ & + \frac{H^4}{32\pi^2}\int dww\Big[\psi\Big(\frac{1}{2}+\nu\Big) + \psi\Big(\frac{1}{2}-\nu\Big)\Big] , \end{aligned} \tag{62}$$

where λ and ξ are now t-dependent according to (59) and (60). We remark that even after the improvement logarithmic terms still appear in the expression for the effective potential. As it can be seen from Eq. (40), these logarithmic terms vanish in the limit when $H \to 0$. Had we introduced the renormalization scale H_0 for the Hubble parameter and then solved the CS equation for this case, we could in principle get rid off all logarithmic terms. However, as we shall see later, for the calculation of the quantum radiative corrections to slow-roll parameters the expression (62) suffices because our final results do not depend on H. With this in mind we use Eq. (62) in the next section. For an alternative approach to the renormalisation group improved scalar effective theories in de Sitter space see Refs. [34, 35].

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[1] The choice (61) is not unique. However, this is the unique choice for which $\varphi_0\partial_{\varphi_0} = -\partial_t$ when $H^2 \to 0$ in the CS equation (52). For any other choice of t there is an additional prefactor in front of ∂_t, and after dividing the CS equation with that prefactor one can solve it *as if* the β functions are modified, which in turn will modify the functions $\lambda(t)$ and $\xi(t)$. This in principle leads to a different effective RG improved theory, which however differs from the one we use here only at higher orders in the coupling constants.

5. SLOW-ROLL PARAMETERS

In this section we calculate the quantum[2] one-loop corrections to the slow-roll parameters ϵ and η arising from the scalar matter vacuum fluctuations in inflation. Within the slow-roll approximation we can drop the kinetic term in the action because it is formally second order in slow-roll parameters [28]. That implies that – within the slow-roll approximation – the leading contribution to the stress-energy tensor is given by[3]

$$T_{\mu\nu} = -g_{\mu\nu}\left(V_{\rm RG}(\varphi) - \frac{1}{4}\frac{\delta V_{\rm RG}(\varphi)}{\delta \ln H}\right)\,, \tag{64}$$

where $V_{\rm RG}(\varphi)$ is the improved renormalized effective potential (62). A straightforward calculation yields,

$$T_{\mu\nu} = -g_{\mu\nu}\left(\frac{\varphi^4}{4!}\,A + 3H^2\varphi^2\,B\right)\,, \tag{65}$$

where we have introduced

$$\begin{aligned} A &\equiv \lambda + \frac{3\lambda^2}{32\pi^2}\left(X - \frac{25}{6}\right)\,, \\ B &\equiv \xi + \frac{\lambda(\xi-\frac{1}{6})}{32\pi^2}\left(X - 4 + \frac{1}{36(\xi-\frac{1}{6})}\right)\,, \end{aligned} \tag{66}$$

and

$$X \equiv \ln\left(\frac{2H^2}{\lambda\varphi^2}\right) + \psi\left(\frac{1}{2}+\nu\right) + \psi\left(\frac{1}{2}-\nu\right)\,. \tag{67}$$

The above result for the stress-energy tensor we insert into the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu} \tag{68}$$

to obtain the following *quantum Friedmann equation*

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{\varphi^4}{4!}A + 3H^2\varphi^2 B\right)\,, \tag{69}$$

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[2] In this work when we refer to the 'classical' value of a parameter we mean its tree-level value. When we refer to the 'quantum correction' we mean the one-loop contribution to the corresponding parameter.

[3] In Eq. (64) we have neglected the tree-level contributions to the stress-energy tensor, which are proportional to $\xi(d/dt)^2(\varphi^2)$ and $\xi H(d/dt)(\varphi^2)$. These terms can be neglected based on the observation that the condition $\dot H \ll H^2$ together with the slow-roll approximation imply

$$3\xi H\varphi\dot\varphi \ll \frac{1}{3}V(\varphi)\,. \tag{63}$$

For a derivation of this condition we refer to Ref. [36].

where $M_{\rm Pl}^2 = 1/(8\pi G_N)$. If we take from A and B, defined in (66), the leading (classical) contributions, then we can from (69) extract the classical Friedmann equation in the form,

$$H_C^2 = \frac{\lambda\varphi^4}{72 M_{\rm Pl}^2}\left(1-\xi\frac{\varphi^2}{M_{\rm Pl}^2}\right)^{-1}, \tag{70}$$

which in the limit $\xi \to 0$ reduces to the well-known result. We shall use equation (70) to calculate the number of e-foldings N in the next section.

In order to determine the slow-roll parameters we still need an expression for $\dot{\varphi}$. From Eq. (62) and the slow-roll form[4] of the scalar field equation,

$$3H\dot{\varphi} + \frac{dV_{\rm RG}}{d\varphi} = 0\,, \tag{71}$$

we obtain

$$\dot{\varphi} = -\frac{W}{3H}\,, \tag{72}$$

where

$$W \equiv \frac{\varphi^3}{3!}\,C + 12H^2\varphi\,D + 72\frac{H^4}{\varphi}\,E\,, \tag{73}$$

and

$$\begin{aligned}
C &\equiv \lambda + \frac{3\lambda^2}{32\pi^2}\left(X - \frac{11}{3}\right) + \frac{9}{4}\frac{\lambda^3}{(4\pi)^4}\left(X - \frac{25}{6}\right)\,,\\
D &\equiv \xi + \frac{\lambda(\xi-\frac{1}{6})}{32\pi^2}\left(X - 3 + \frac{1}{36(\xi-\frac{1}{6})}\right) + \frac{\lambda^2(\xi-\frac{1}{6})}{(4\pi)^4}\left(X - \frac{15}{4} + \frac{1}{48(\xi-\frac{1}{6})}\right)\,,\\
E &\equiv \frac{\lambda(\xi-\frac{1}{6})^2}{(4\pi)^4}\left[\psi\left(\frac{1}{2}+\nu\right) + \psi\left(\frac{1}{2}-\nu\right)\right]\,.
\end{aligned} \tag{74}$$

Upon inserting the leading contributions from Eq. (74) into Eq. (72), the classical expression for $\dot{\varphi}$ follows immediately,

$$\dot{\varphi}_C = -\frac{1}{3H_C}\left(\frac{\lambda\varphi^3}{6} + 12\xi H_C^2\varphi\right)\,. \tag{75}$$

It is important to note that with the above definitions

$$\begin{aligned}
C &= A + \frac{1}{4}\beta_\lambda + \mathcal{O}(\lambda^3)\,,\\
D &= B + \frac{1}{2}\beta_\xi + \mathcal{O}(\lambda^2)\,.
\end{aligned} \tag{76}$$

We keep the parameter E in (73) for completeness, although it yields only higher order contributions comparing to other parameters defined by (66) and (74).

[4] Within the slow-roll approximation we can drop the $\ddot{\varphi}$ term in Eq. (71) because that term is second order in slow-roll parameters.

We now proceed by making use of the standard result for the spectrum of primordial curvature perturbation [28]

$$P_{\mathcal{R}}(k) = \left[\left(\frac{H}{\dot{\varphi}} \right)^2 \left(\frac{H}{2\pi} \right)^2 \right] \Bigg|_{k=aH} . \tag{77}$$

In deriving this expression the canonical quantization of the inflaton field has been performed in the standard way, by studying the evolution of small perturbations around the inflaton condensate. Since in our approach the quantum corrections are calculated at the level of the effective potential, which changes the on-shell structure of the theory but does not change the structure of Eq. (77), we *conjecture* that Eq. (77) can be used without any further generalization for the calculation of the one-loop quantum corrections to the spectrum of curvature perturbation and the implied slow-roll parameters that arise from the matter vacuum fluctuations within the framework proposed in this work. A proof of this conjecture is nevertheless desirable. The right-hand side of (77) is evaluated at the horizon exit, at which $k = aH$, because during slow-roll inflation the Hubble parameter does not change significantly over a few Hubble times [28].

The scalar spectral index n_s is defined as

$$n_s - 1 = \frac{d \ln P_{\mathcal{R}}}{d \ln k} , \tag{78}$$

which after some algebra, by making use of (77) and (72), yields

$$n_s - 1 = -\frac{W}{H^4} \frac{dH^2}{d\varphi} + \frac{2}{3H^2} \frac{dW}{d\varphi} . \tag{79}$$

By analogy with the standard result

$$n_s - 1 = -6\epsilon + 2\eta , \tag{80}$$

which is valid at the classical level for various inflationary models, we define

$$\begin{aligned} \epsilon &\equiv \frac{W}{6H^4} \frac{dH^2}{d\varphi} , \\ \eta &\equiv \frac{1}{3H^2} \frac{dW}{d\varphi} , \end{aligned} \tag{81}$$

such that equation (80) still holds for the quantum case. On the other hand, for the gravitational wave spectrum we use the result

$$P_g = \frac{8}{M_{\rm Pl}^2} \left(\frac{H}{2\pi} \right)^2 \Bigg|_{k=aH} . \tag{82}$$

The gravitational wave spectral index n_g is defined as

$$n_g = \frac{d\ln P_g}{d\ln k}\,, \tag{83}$$

from which it follows that within our framework,

$$n_g = -\frac{W}{3H^4}\frac{dH^2}{d\varphi}\,. \tag{84}$$

After taking into account the definition of ϵ from (81) we obtain that the standard result,

$$n_g = -2\epsilon\,, \tag{85}$$

remains valid for the quantum case as well. It is also convenient to introduce the ratio r between the gravitational wave spectrum and the spectrum of primordial curvature perturbation,

$$r \equiv \frac{P_g}{P_{\mathcal{R}}}\,, \tag{86}$$

which here turns into

$$r = \frac{8}{9M_{\rm Pl}^2}\frac{W^2}{H^4}\,. \tag{87}$$

With the above definitions the standard relation, $r = 16\epsilon$ is not any more satisfied at the quantum level. However, we still expect to reproduce it at the classical limit (but see the discussion below).

The final result for ϵ and η, which includes both the classical and quantum contributions, we write in the form

$$\begin{aligned} \epsilon &= \epsilon_C + \epsilon_Q\,, \\ \eta &= \eta_C + \eta_Q, \end{aligned} \tag{88}$$

and we separate the quantum contributions into the following two characteristic parts,

$$\begin{aligned} \epsilon_Q &= \frac{\beta_\lambda}{\lambda}\,Q_{\epsilon\lambda} + \beta_\xi\,Q_{\epsilon\xi}\,, \\ \eta_Q &= \frac{\beta_\lambda}{\lambda}\,Q_{\eta\lambda} + \beta_\xi\,Q_{\eta\xi}\,. \end{aligned} \tag{89}$$

Although the two contributions are formally of the same order of magnitude, they have a different origin. The former contribution in Eq. (89) arises as a result of the resummation of the mass insertions $m_\varphi^2 = \lambda\varphi^2/2$ generated by the quartic self-interaction in the presence of an inflaton condensate. The latter contribution is a consequence of the resummation induced by the effective mass parameter $12\xi H^2$ generated by the inflaton field coupled to the background curvature. Now we shall present our results, first the classical part.

5.1. Classical contributions ϵ_C and η_C

After a straightforward calculation, we arrive at

$$\begin{aligned}
\epsilon_C &= \frac{8}{z}\frac{1-\frac{1}{2}\kappa}{1-\kappa}\,, \\
\eta_C &= \frac{12}{z}\frac{1-\frac{1}{3}\kappa}{1-\kappa}\,,
\end{aligned} \tag{90}$$

where z and κ are defined by

$$\begin{aligned}
z &\equiv \frac{\varphi^2}{M_{\rm Pl}^2}\,, \\
\kappa &\equiv \xi z = \xi \frac{\varphi^2}{M_{\rm Pl}^2}\,.
\end{aligned} \tag{91}$$

It is clear that in the limit when $\xi \to 0$, i.e. when $\kappa \to 0$, we recover the standard expressions for the slow-roll parameters in the $\lambda\varphi^4$ inflationary model; namely $\epsilon = 8M_{\rm Pl}^2/\varphi^2$ and $\eta = 12M_{\rm Pl}^2/\varphi^2$. This is not surprising since in this limit our theory reduces precisely to that inflationary model.

We now introduce the number of e-foldings

$$N = -\int_{t_{\rm end}}^{t} H dt\,, \tag{92}$$

which somewhat surprisingly, when calculated classically ($H \to H_C$), gives the same result as the $\lambda\varphi^4$ inflationary model

$$N = \frac{1}{M_{\rm Pl}^2}\left(\varphi^2 - \varphi_{\rm end}^2\right)\,. \tag{93}$$

However, a mild ξ-dependence does enter the expression for N through the value of the inflaton field at the end of inflation, $\varphi_{\rm end}$, which is determined from the condition $\epsilon_C(\varphi_{\rm end}) = 1$. From (90) it follows[5]

$$\varphi_{\rm end}^2 \simeq 4M_{\rm Pl}^2(2-\xi), \tag{94}$$

and finally

$$z \equiv \frac{\varphi^2}{M_{\rm Pl}^2} = 8\tilde{N}\,, \qquad \tilde{N} \simeq N + 1 - \frac{1}{2}\xi\,. \tag{95}$$

We shall use the above notation when writing the quantum contributions to slow-roll parameters.

[5] The result (94) is valid to the leading order in ξ.

5.2. Quantum contributions ϵ_Q and η_Q

In calculating the quantum contributions to slow-roll parameters we must take into account the observational constraint required by the near scale invariance of the spectrum

$$\left|\frac{1}{2}\lambda\varphi^2 + 12\xi H^2\right| \ll H^2 \,. \tag{96}$$

In view of equations (41)-(48) the observational constraint (96) implies that, in order to study the quantum radiative corrections to slow-roll parameters, we need the infrared limit of the RG improved theory (62). This is the opposite limit from the ultraviolet limit in which our effective theory reduces to the Coleman-Weinberg form (40) [27]. That means that in order to study the quantum radiative corrections to slow-roll parameters, one needs to focus on the infrared radiative corrections which are specific for (quasi-)de Sitter spaces, and completely absent in Minkowski space (which is related by a conformal rescaling to our $n = 0$ case), and hence has a very different infrared structure. In particular, the most singular term $a_n/\delta w$ in (48) is absent in the conformal $n = 0$ case ($a_0 = 0$). In summary, that means that the infrared sector plays a crucial role in determining the quantum corrections to slow-roll parameters.

After taking into account the constraint (96) and the expression (70) for the classical Friedmann equation, we arrive at the condition

$$\left|\frac{9}{2\tilde{N}} - 24\xi\right| \ll 1 \,, \tag{97}$$

Together with the condition $8\tilde{N}\xi < 1$, this equation then gives,

$$-\frac{1}{24}\left(1 - \frac{9}{2\tilde{N}}\right) \ll \xi < \frac{1}{8\tilde{N}} \,. \tag{98}$$

Recall that typically N (and hence also $\tilde{N}$) is between 50 and 60 such that the term $9/(2\tilde{N}) \sim 10^{-1} \ll 1$ in Eq. (98) can be to a good approximation neglected.

We proceed by writing approximately the relation (67) as ($\delta w \ll 1$)

$$X = \ln\left(\frac{\lambda\varphi^2}{2H^2}\right) - \frac{3}{\delta w} - 2\gamma_E + \frac{7}{3} + \mathcal{O}(\delta w) \,, \tag{99}$$

from which it follows

$$\frac{dX}{d\varphi} = -\frac{1}{H^2}\frac{dH^2}{d\varphi}\,\zeta + \frac{2}{\varphi}\,\zeta \,, \tag{100}$$

where we have introduced

$$\zeta \equiv -\left(1 + \frac{3}{\delta w^2}\frac{\lambda\varphi^2}{2H^2}\right) \,. \tag{101}$$

Now the calculation of the slow-roll parameters ϵ and η is straightforward; here we present only our final results. Some intermediate steps and results can be found in Appendix A.

For the quantum contribution (89) to the slow-roll parameter ϵ we obtain

$$\begin{aligned}
Q_{\epsilon\lambda} &= \frac{5}{27}\frac{1-\frac{29}{15}\kappa+\frac{17}{18}\kappa^2}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2}+\frac{1}{z}\frac{6-5\kappa+\kappa^2}{1-\kappa}\,, \\
Q_{\epsilon\xi} &= -\frac{z}{18}\frac{\kappa}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2}+\frac{2}{(1-\kappa)^2}\ln\left(\frac{z}{1-\kappa}\right)+4\,\frac{2-\sigma-2\kappa+\frac{1}{2}\kappa^2}{(1-\kappa)^2}\,,
\end{aligned} \tag{102}$$

and for the η slow-roll parameter defined in (89)

$$\begin{aligned}
Q_{\eta\lambda} &= \frac{10}{27}\frac{1-\frac{29}{15}\kappa+\frac{17}{18}\kappa^2}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2}+\frac{1}{z}\frac{11-10\kappa+3\kappa^2}{1-\kappa} \\
&= 2Q_{\epsilon\lambda}-\frac{1+\kappa}{z}\,, \\
Q_{\eta\xi} &= -\frac{z}{9}\frac{\kappa}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2}+\frac{4}{(1-\kappa)^2}\ln\left(\frac{z}{1-\kappa}\right)+\frac{18-8\sigma-20\kappa+6\kappa^2}{(1-\kappa)^2} \\
&= 2Q_{\epsilon\xi}+2\,.
\end{aligned} \tag{103}$$

In Eqs. (102) and (103) we abbreviated

$$\begin{aligned}
z &\equiv \frac{\varphi^2}{M_{\rm Pl}^2} = 8\tilde{N}\,, \\
\kappa &\equiv z\xi\,, \\
\sigma &= \frac{5}{6}+\gamma_E+\ln 6\,.
\end{aligned} \tag{104}$$

Since typically the number of e-foldings required during inflationary epoch ranges between 50 and 60, it is evident from (104) that z is of the order 5×10^2, which justifies the ordering of the quantum corrections in powers of z. The leading contributions are the terms linear in z, and they are present only in $Q_{\epsilon\xi}$ and $Q_{\eta\xi}$ in (102) and (103). Both $Q_{\epsilon\xi}$ and $Q_{\eta\xi}$ contain also the next-to-leading terms of the order $\ln(z)$. The (subleading) terms of the order z^0 are in fact the leading contributions to $Q_{\epsilon\lambda}$ and $Q_{\eta\lambda}$ in (102) and (103).

5.3. Tensor and scalar spectral indices n_g and n_s

We can now easily calculate the tensor and scalar spectral indices from the results for the slow-roll parameters ϵ and η (90), (102–103).

Note first that the tensor spectral index n_g (85) can be trivially obtained by summing the classical (90) and quantum (102) contributions for ϵ, since from our definition (81) it follows that Eq. (85) is valid also at the quantum level.

Next we consider the scalar spectral index n_s. By making use of Eqs. (90) and (102–103) and separating again the classical and quantum contributions as,

$$n_s - 1 = (n_s - 1)_C + (n_s - 1)_Q \,, \tag{105}$$

where

$$(n_s - 1)_Q = \frac{\beta_\lambda}{\lambda}\, Q_{(n_s-1)_\lambda} + \beta_\xi\, Q_{(n_s-1)_\xi} \,, \tag{106}$$

we arrive at the classical scalar spectral index,

$$(n_s - 1)_C = -\frac{24}{z}\,\frac{1-\frac{2}{3}\kappa}{1-\kappa} \,, \qquad z = 8\tilde{N} \,, \qquad \kappa = 8\tilde{N}\xi \,. \tag{107}$$

The quantum contributions are given by,

$$\begin{aligned} Q_{(n_s-1)_\lambda} &= -\frac{10}{27}\,\frac{1-\frac{29}{15}\kappa+\frac{17}{18}\kappa^2}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2} - \frac{1}{z}\,\frac{14-10\kappa}{1-\kappa} \,, \\ Q_{(n_s-1)_\xi} &= \frac{z}{9}\,\frac{\kappa}{(1-\kappa)^2(1-\frac{2}{3}\kappa)^2} - \frac{4}{(1-\kappa)^2}\ln\left(\frac{z}{1-\kappa}\right) - \frac{12-8\sigma-8\kappa}{(1-\kappa)^2} \,. \end{aligned} \tag{108}$$

Notice that the quantum contributions $(\beta_\lambda/\lambda)Q_{(n_s-1)_\lambda}$ and $\beta_\xi Q_{(n_s-1)_\xi}$ are both much smaller than the classical contribution $(n_s-1)_C$ due to the fact that both β_λ/λ and β_ξ are of the order of λ, which is constrained by experimental data to be of the order of $\lambda \sim 10^{-12}$ (we provide a more precise constraint for λ below). The quantum contributions can become significant only in an inflationary model in which the relevant coupling constant can be as large as the order of 10^{-3}. It is important to notice that $Q_{(n_s-1)_\lambda}$ provides an infrared enhanced quantum correction proportional to the number of e-foldings N, while, on the other hand, $Q_{(n_s-1)_\xi}$ contains a correction which is enhanced by N^2 when compared to the naïve expectation $\mathcal{O}(1/z) \sim 1/N$).

This shows that in the de Sitter invariant limit the quantum corrections to slow-roll parameters accumulate only from the time the mode becomes super-Hubble until the end of inflation. For a mode which exits horizon N e-foldings before the end of inflation, the whole history of inflation before the Hubble exit (*i.e.* when the mode was sub-Hubble) is completely irrelevant and does not contribute in a cumulative manner to the quantum corrections. This disagrees with the result found in Refs. [12, 13], where it is claimed that the quantum loop corrections induce corrections to the slow-roll parameters and spectral indices which depend on the total duration of inflation and are thus enhanced when inflation lasts for a large number of e-foldings. This N^2 enhancement is the main result of our work, and it resolves the Weinberg's dilemma [10, 11]: how big can be the correction induced by the quantum fluctuations of light or massless scalar fields during inflation,

given the fact that the (equal time and space) scalar field correlator for a massless minimally coupled scalar grows linearly with time during (de Sitter) inflation,

$$\langle 0|\, \varphi^2(x)\, |0\rangle = \text{infinite} + \frac{H^2}{4\pi^2}\ln a\,, \tag{109}$$

or

$$\big(\langle 0|\, \varphi^2(x)\, |0\rangle\big)\big|_{\rm fin} \simeq \frac{H^2}{4\pi^2}N\,, \tag{110}$$

where here N denotes (minus) the number of e-foldings.

By comparing Eqs. (85) (102) and (108) we observe that the following curious relation holds,

$$(n_g)_Q = (n_s-1)_Q + \frac{\beta_\lambda}{\lambda}\left(\frac{2(1+\kappa)}{z}\right) - 4\beta_\xi\,, \tag{111}$$

such that the leading quantum contributions $\mathcal{O}(\lambda\tilde{N})$ and $\mathcal{O}(\lambda\ln(\tilde{N}))$ to the tensor and scalar spectral indices are equal. This approximate equality can be traced back to the fact that $\epsilon_Q \simeq \eta_Q/2$, from where it follows that $[(W/H^4)(dH^2/d\varphi)]_Q \simeq [(1/H^2)(dW/d\varphi)]_Q$, or equivalently $[d(W/H^2)/d\varphi]_Q \simeq 0$. The expression W/H^2 is proportional to the square-root of the ratio $r = P_g/P_{\mathcal{R}}$, which according to Eq. (121) does not receive any quantum corrections that are amplified by N. That means that, because the tensor and scalar spectra are identically affected by the quantum corrections enhanced by N, these leading corrections cancel in the ratio r. We do not have a deeper insight to why that is the case. Note that the approximate equality (111) does not hold for the corresponding classical parts (90) and (107). If inflationary models with large quantum corrections are found, Eq. (111) could be used to resolve the quantum from classical contributions to the tensor and scalar spectral indices.

5.4. The spectrum of curvature perturbation $\mathcal{P}_{\mathcal{R}}$ and the tensor-to-scalar ratio r

Next we consider the spectrum of curvature perturbation (77). As usual we decompose the spectrum into the classical and quantum parts as,

$$P_{\mathcal{R}} = (P_{\mathcal{R}})_C + (P_{\mathcal{R}})_Q\;, \tag{112}$$

where

$$(P_{\mathcal{R}})_Q \equiv \frac{\beta_\lambda}{\lambda}Q_{P_{\mathcal{R}}\lambda} + \beta_\xi Q_{P_{\mathcal{R}}\xi}\,. \tag{113}$$

Working within our framework we obtain,

$$(P_{\mathcal{R}})_C = \frac{\lambda}{9\pi^2}\frac{\tilde{N}^3}{1-\kappa}\,, \tag{114}$$

and

$$\frac{Q_{P_{\mathcal{R}}\lambda}}{(P_{\mathcal{R}})_C} = -\frac{z}{27}\frac{1-\frac{25}{24}\kappa}{(1-\kappa)(1-\frac{2}{3}\kappa)} + \frac{1}{2}\ln\left(\frac{z}{1-\kappa}\right) - \frac{1}{2}(1-\kappa) - \sigma - \frac{1}{12},$$
$$\frac{Q_{P_{\mathcal{R}}\xi}}{(P_{\mathcal{R}})_C} = -\frac{z^2}{24}\frac{1}{(1-\kappa)(1-\frac{2}{3}\kappa)} + \frac{z}{2}\frac{1}{1-\kappa}\ln\left(\frac{z}{1-\kappa}\right) - z\left(\frac{\sigma}{1-\kappa}+1\right), \tag{115}$$

where $\tilde{N} = N+1-\frac{1}{2}\xi$, $z = 8\tilde{N}$ and $\kappa = 8\xi\tilde{N}$. This implies that, just like the spectral indices, when compared to the classical contribution, the quantum contribution to the spectrum is suppressed as $\lambda\tilde{N}^2$, and thus unobservably small for the model in consideration.

The spectrum of curvature perturbation is an observable quantity and the three year WMAP data provide a strong constrain for it [2],

$$P_{\mathcal{R}} \approx 29.5 \times 10^{-10} A\,, \qquad A = 0.801^{+0.043}_{-0.054}\,. \tag{116}$$

In the case of a weak coupling to the background, *i.e.* when $8|\xi|\tilde{N} \ll 1$ and $\tilde{N} \approx N+1$, Eqs. (114) and (116) imply

$$\lambda_{50} \approx 1.58 \times 10^{-12}\,, \qquad \lambda_{60} \approx 9.25 \times 10^{-13}\,, \qquad (8|\xi|\tilde{N} \ll 1)\,, \tag{117}$$

where the subscripts on λ denote the number of e-foldings N. We stress that these values for λ are strictly speaking valid only for this particular model. Indeed, when we choose the coupling to the background in the range,

$$\frac{1}{24} \gg -\xi > \frac{1}{8\tilde{N}}\,, \tag{118}$$

then from Eq. (114) we see that the value of λ can be up to one order of magnitude larger,

$$\lambda_{50} \simeq 2.69 \times 10^{-11} \times (-24\xi)\,, \qquad \lambda_{60} \simeq 1.88 \times 10^{-11} \times (-24\xi)\,, \qquad (-24\xi \ll 1)\,. \tag{119}$$

These larger values of λ are still too small however to render the quantum effects observable.

In other inflationary models the relation for the spectrum of curvature perturbation (114) can in general be different, hence allowing for models in which couplings are larger. Furthermore, the quantum effects in some other models may be much stronger – of particular interest are hybrid inflationary models [1].

Let us now consider the ratio r of the gravitational wave and curvature spectrum. From Eq. (87) we obtain

$$r = r_C + r_Q$$
$$r_C = \frac{128}{z} \tag{120}$$
$$r_Q = \frac{\beta_\lambda}{\lambda}\frac{64}{z}(1-\kappa) + 128\beta_\xi\,, \tag{121}$$

where again $z \equiv 8\tilde{N}$ and $\kappa \equiv z\xi$. We see that $r_C = 128/z$, and after observing from (90) that $\epsilon_C = 8/z$ in the limit when $\xi \to 0$ (i.e. when $\kappa \to 0$), we obtain the standard result $r = 16\epsilon$. This relation is violated both by the classical corrections from $\kappa \neq 0$ ($\xi \neq 0$) and by the quantum contributions. In particular, when $\kappa = 8\tilde{N}\xi \ll -2$, one gets $r \simeq 32\epsilon$. When compared with the classical contribution (120), the quantum contribution to r (121) is as usual suppressed by λ, but – in contrast to the slow-roll parameters ϵ_Q and η_Q – it is not enhanced by powers of N.

It is well known that the minimally coupled $\lambda\varphi^4$ inflationary model is disfavored by observations [2] by about 2 standard deviations. This is not however in general the case with the non-minimally coupled $\lambda\varphi^4$ inflationary model. Indeed, from Eqs. (98) and (107) we infer that in the range

$$\frac{1}{24} \gg -\xi > \frac{1}{8\tilde{N}} \,. \tag{122}$$

the deviation of the classical spectral index of scalar curvature perturbation (107) from scale invariance is reduced approximately by a factor of 2/3,

$$(n_s - 1)_C \approx -\frac{16}{z} = -\frac{2}{\tilde{N}} \,, \qquad (-\xi \gg (8\tilde{N})^{-1}) \,, \tag{123}$$

while the value of r_C (120) remains unchanged. This then implies that, similarly as in the minimally coupled massive inflationary model, the $\lambda\varphi^4$ model with ξ in the range (122) falls roughly at the 1σ contour of Figure 14 in Ref. [2], rendering these nonminimally coupled $\lambda\varphi^4$ inflationary models consistent with the three year WMAP data [2, 4, 5, 6]. We emphasize that, because $\xi = \xi(\varphi_0)$ runs logaritmically towards its infrared fixed point $\xi_{\rm FP} = 1/6$, it is natural to assume that ξ deviates from zero. Indeed, even if we choose $\xi = 0$ (which corresponds to the coupling at some scale $\varphi = \varphi_0$), the running of ξ will induce the dominant quantum contributions to slow-roll parameters. Thus for consistency it is necessary to consider the effects of nonminimal coupling, and choosing ξ in the range (122) is *a priory* as natural as any other choice (different, of course, from $\xi = 1/6$).

In conclusion, we have found out that, even though the quantum effects to slow-roll parameters are enhanced by the number of e-foldings squared, they are suppressed by the small coupling constant λ. Due to the smallness of λ however, the quantum effects have a negligible impact on the plot presented for example in Figure 14 of [2].

6. DISCUSSION

In this work we develop a quantum field theoretic framework within which the quantum corrections to slow-roll parameters and observables from inflationary models can be calculated. Whe main

purpose of this paper is methodical, and we postpone a detailed study of the quantum corrections to inflationary observables in various inflationary models to a future work [1].

We illustrate how our framework works by performing the relevant calculations in a concrete inflationary model chosen for its simplicity. More specifically, we consider a $\lambda\varphi^4$ inflationary model (13–15) with a nonminimal coupling to the background curvature. Our formalism can be quite straightforwardly generalized to other inflationary models. Within our $\lambda\varphi^4$ model we calculate the quantum corrections to the inflationary slow-roll parameters ϵ and η (89), (102–103), based on which we derive the scalar spetral index n_s (105–108), the tensor spectral index n_g (85), (103) and the spectrum of curvature perturbation $P_{\mathcal{R}}$ (112–115). These corrections arise from the one-loop scalar vacuum fluctuations during de Sitter inflation. We find that the dominant quantum effects for the spectral indices (2), (85), (103) are suppressed as λN^2 when compared to the classical (tree-level) results (3), (90). The dominant quantum contribution arises from the inflaton coupling to the background curvature.

Our theoretic framework can be improved in several aspects. For example, one could generalize our calculation of the renormalization group improved effective action (62) to quasi-de Sitter spaces, which are important since these spaces comprise a large fraction of inflationary models. Next one should generalize our analysis to incorporate other matter degrees of freedom which would allow us to incorporate a broad spectrum of inflationary models. Even more importantly one should study the role of the interactions that couple matter and gravitational degrees of freedom.

Appendix A

Here we provide a more detailed derivation for the quantum contributions to slow-roll parameters given by the relations (102) and (103). We begin by abbreviating the equalities in Eq. (66) as

$$\begin{aligned} A &= \lambda + \beta_\lambda X_A\,, \\ B &= \xi + \beta_\xi X_B\,, \end{aligned} \tag{124}$$

where now

$$\begin{aligned} X_A &\equiv \frac{1}{2}\left(X - \frac{25}{6}\right)\,, \\ X_B &\equiv \frac{1}{2}\left(X - 4 + \frac{1}{36\left(\xi - \frac{1}{6}\right)}\right)\,. \end{aligned} \tag{125}$$

It is important to note that, in order to strictly follow our notation in Eq. (89), the last term in the definition of X_B actually contributes to both $Q_{\epsilon\lambda}$ and $Q_{\eta\lambda}$. The reason is that, after taking into

account the relations (54) for β_λ and β_ξ, it follows immediately that,

$$\beta_\xi \frac{1}{72\left(\xi - \frac{1}{6}\right)} = \frac{1}{72}\frac{\lambda}{16\pi^2} = \frac{1}{216}\frac{\beta_\lambda}{\lambda}\,. \tag{126}$$

The origin of this mixing is the peculiar 1/36 term in (62), which is suppressed by λ, but not by $\xi - \frac{1}{6}$.

It is convenient to introduce the parameter α as follows,

$$\alpha \equiv \left(\frac{M_{\rm Pl}^2}{\varphi^2} - \xi\right)^{-1}, \tag{127}$$

which, from the definitions in (91), can also be expressed as

$$\alpha = \frac{z}{1-\kappa}\,. \tag{128}$$

Assuming that $\xi\varphi^2/M_{\rm Pl}^2 < 1$ is far enough from 1, from the quantum Friedmann equation (69) we obtain the following expression,

$$\frac{\varphi^2}{H^2} \simeq \frac{72}{\alpha\lambda}\left(1 - \beta_\xi \alpha X_B - \frac{\beta_\lambda}{\lambda}X_A\right)\,. \tag{129}$$

Upon differentiating the quantum Friedmann equation (69) with respect to φ we get,

$$\frac{dH^2}{d\varphi} = \frac{\alpha\varphi}{18}\frac{\lambda + \beta_\lambda\zeta_A + 36\frac{H^2}{\varphi^2}\left(\xi + \beta_\xi\zeta_B\right)}{1 + \beta_\lambda\frac{\varphi^2}{H^2}\frac{\zeta\alpha}{144} + \beta_\xi\alpha\left(\frac{\zeta}{2} - X_B\right)}\,. \tag{130}$$

In deriving this expression we have used,

$$\begin{aligned}\frac{dA}{d\varphi} &\simeq \frac{1}{\varphi}\beta_\lambda + \frac{1}{2}\frac{dX}{d\varphi}\beta_\lambda\,,\\ \frac{dB}{d\varphi} &\simeq \frac{1}{\varphi}\beta_\xi + \frac{1}{2}\frac{dX}{d\varphi}\beta_\xi\,,\end{aligned} \tag{131}$$

where $dX/d\varphi$ and ζ are given by Eqs. (100) and (101), respectively. In writing Eq. (130) we have also introduced

$$\begin{aligned}\zeta_A &\equiv X_A + \frac{1}{4}(1+\zeta)\,,\\ \zeta_B &\equiv X_B + \frac{1}{2}(1+\zeta)\,.\end{aligned} \tag{132}$$

In order to evaluate ϵ from (81) we still need the expression for W. With the above definitions, from (73) it follows

$$W \simeq \frac{\lambda\varphi^3}{6}\left[1 + \alpha\xi + \frac{\beta_\lambda}{\lambda}\left(\frac{1}{4} + X_A(1+\alpha\xi)\right) + \beta_\xi\alpha\left(\frac{1}{2} + X_B(1+\alpha\xi)\right)\right]\,. \tag{133}$$

What remains to be done is to expand the denominator in (130) to the linear order in β_λ and β_ξ and then insert the resulting expression together with (129) and (133) into the definition of ϵ given in (81). In this manner both the classical (90) and the quantum contributions (102) can be obtained.

To calculate η, we must determine $dW/d\varphi$. After some algebra we arrive at

$$\begin{aligned} \frac{dW}{d\varphi} &\simeq \frac{dH^2}{d\varphi} 12\varphi \left[\xi - \frac{\beta_\lambda}{\lambda}\frac{\zeta}{2\alpha} + \beta_\xi \left((X_B + \frac{1}{2}) - \frac{1}{2}\zeta \right) \right] \\ &\quad + \frac{1}{2}\varphi^2 \left[\lambda + \beta_\lambda \left(X_A + \frac{\zeta}{3} + \frac{7}{12} \right) \right] \\ &\quad + 12H^2 \left[\xi + \beta_\xi \left(X_B + \zeta + \frac{3}{2} \right) \right] , \end{aligned} \tag{134}$$

where $dH^2/d\varphi$ is given in (130). After expanding the denominator in (130) and after inserting (129) and (134) into the definition for η given by (81), both the classical (90) and quantum contribution (103) are obtained.

At the end, we summarize

$$\begin{aligned} X_A &\simeq \frac{1}{2}\left(\ln\alpha - \frac{3}{\delta\omega} \right) - \sigma - \frac{1}{12} , \\ X_B &\simeq \frac{1}{2}\left(\ln\alpha - \frac{3}{\delta\omega} + \frac{1}{36\left(\xi - \frac{1}{6}\right)} \right) - \sigma , \\ \delta w &\simeq \frac{36}{\alpha} + 12\xi , \\ \zeta &\simeq - \left[1 + \frac{3}{4}\frac{\alpha}{(3+\alpha\xi)^2} \right] , \end{aligned} \tag{135}$$

where α is determined by (128), while σ is given by (104).

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